ABRHS Physics (H) Name: KEY

Geometric Units

We have already pointed out how convenient units can make calcalations eaiser when it was suggested to measure distances in "light-years" or "light-seconds" while also using "years" or "seconds" to measure time. In this way, the speed of light is by definition 1 ly/y or 1 ls/s, and all the c^2 terms in the equations mostly go away. One should still write out the c^2 terms in the equations so that the units work out correctly, but once the numbers are plugged in, the math is a lot easier.

It turns out, we can take this idea one step further. In the SI system, the speed of light in a vacuum is defined as *exactly* 299,792,458 m/s. The meter is then defined as 1/299,792,458 the distance light travels in 1 second. The second is then defined as a the time for a certain number of a particular atomic oscillation. Meters and seconds are not independent of each other – their ratio is fixed by the universal consant of the speed of light in a vacuum.

Now for the big idea. Let's make a new set of units and we will start by defining the speed of light to be 1.

$$c = 1$$
$$(1 = 3 \times 10^8 \text{ m/s})$$

Notice how there are no units. Speed in this new system is a dimensionless quantity. To help your transition to this, just think of speeds are now given as a fraction of the speed of light. Not very helpful for posting speed limits, but certainly convenient when doing relativity. Here is where it gets odd for people: distance and time use the same unit! (Otherwise, distance/time would not be dimensionless.) We will use meters to measure both distance and time. Welcome to the world of Geometric Units!

In geometric units, meters for distance are just like you already know and love. The lab benches are still 1.52 meters wide. But what does it mean if you say "I will see in 2 meters?" We can convert back to SI units using the speed of light. Since $c = 1 = 3 \times 10^8$ m/s, we can say $(2 \text{ m})/(3 \times 10^8 \text{ m/s}) = 6.7 \times 10^{-9}$ s. So "2 meters" of time is 6.7 ns. What is the advantage to this? Because c is just 1, it disappears from the equations.

But wait – there's more! Since general relativity deals with gravity, the universal gravitational constant G shows up all over the place. We will use meters to measure mass as well by defining

$$G = 1$$
$$(1 = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)$$

usually written as $\frac{N \cdot m^2}{kg^2}$ →

$\frac{m^3}{kg\, s^2} = \frac{\frac{kg \cdot m}{s^2} \cdot m^2}{kg^2} = \frac{N \cdot m^2}{kg^2}$

Again, this all seems weird when thinking about daily life, but when doing relativity work, it makes the equations simpler to write and easier to work with. Converting between meters and seconds only involves the speed of light. If there are kilograms involved, the conversion also includes G.

Problems * These are basically dimensional analysis!

1. How many meters is 1 second?

$$(1\ \cancel{s})\left(3 \times 10^8 \frac{m}{\cancel{s}}\right) = \boxed{3 \times 10^8\ m}$$

2. What is a speed of 30 m/s in geometric units?

$$\frac{(30\ m/s)}{(3 \times 10^8\ m/s)} = \boxed{1 \times 10^{-7}}$$

kg cancel

s cancel

Geometric Units

3. What are the geometric units for energy?

$$1\,J = 1\,\frac{kg \cdot m^2}{s^2} \rightarrow \left(\frac{1\,kg \cdot m^2}{s^2}\right)\left(\frac{6.67\times10^{-11}\,m^3}{kg \cdot s^2}\right)\left(\frac{1\,s}{3\times10^8\,m}\right)^4 = \boxed{8.2\times10^{-45}\,m}$$

So units are $\boxed{m}$

4. What are the geometric units for momentum?

just did units! →

$$\left(\frac{kg \cdot m}{s}\right)\left(\frac{m^3}{kg \cdot s^2}\right)\left(\frac{s}{m}\right)^3 = \frac{m^4}{m^3} = \boxed{m}$$

5. If you are 18 years old, what is that in meters?

$$(18\,yrs)\left(\frac{365\,day}{yr}\right)\left(\frac{24\,h}{day}\right)\left(\frac{3600\,s}{h}\right) = 5.68\times10^8\,s$$

So $(5.68\times10^8\,s)\left(\frac{3\times10^8\,m}{s}\right) = \boxed{1.7\times10^{17}\,m}$

6. In m/s, how fast is a speed of 0.2?

$$(0.2)\left(3\times10^8\,\frac{m}{s}\right) = \boxed{6\times10^7\,m/s}$$

7. How many seconds is 5000 m?

$$(5000\,m)\left(\frac{1\,s}{3\times10^8\,m}\right) = \boxed{1.7\times10^{-5}\,s}$$

s cancel m cancel

8. How many kg is 1 m?

$$(1\,m)\left(\frac{1\,kg \cdot s^2}{6.67\times10^{-11}\,m^3}\right)\left(\frac{3\times10^8\,m}{s}\right)^2 = \boxed{1.3\times10^{27}\,kg}$$

9. Can I get back points I lost in September when I used meters to give a time?

No.

Answers: